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COSMIC RAY PROPAGATION

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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Cosmic Ray Propagation

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ABSTRACT

The general problem of the propagation of cosmic rays from their source until they are observed in the vicinity of the earth is reviewed. The effect of matter on the cosmic ray composition is considered in detail and the effect of diffusion is treated in the depth necessary to study some models of the cosmic radiation not previously considered in this manner. The results of these calculations show that there are some aspects of the experimental observations, such as the light to medium nuclei ratio as a function of energy, which are not consistent with any equilibrium model of the cosmic radiation which does not include a rigidity dependence in the mean free path. This conclusion is independent of the source spectrum assumed and is valid for a large class of source distributions. A series of possible explanations for the discrepancy between the observed experimental data and the theoretical predictions of the equilibrium picture are considered, and shown to be unlikely. Faced with this dilemma, attention is turned to a simple non-equilibrium model, consisting of one fairly close single source superimposed on a general background. This picture was chosen since it seemed the simplest first step from equilibrium. The calculations show that this picture leads to considerably better agreement with the experimental data, especially the light to medium nuclei ratio as a function of energy/nucleon. At least one irreconcilable piece of experimental data remains, namely the one measurement of the Fluorine to Oxygen ratio.

I. INTRODUCTION

The origin and history of the energetic cosmic rays which continually bombard the earth is a subject of considerable interest because of its relation to so many fundamental aspects of the galaxy and possibly the universe, including the energy content, the interrelationship of matter, magnetic fields, and cosmic rays, and the nature of the objects which are able to accelerate particles to such high energies in very great numbers. In attempting to obtain a fuller understanding of these questions, one of the fundamental problems which must be examined is the propagation of the cosmic rays from their source to the vicinity of the earth where they are observed.

It is generally believed that after leaving their sources, cosmic rays diffuse through interstellar space with their motion being controlled and made random on a large scale by the magnetic fields. In their passage through interstellar material at least two mechanisms affect the cosmic-ray composition and energy spectra; these are fragmentation produced in nucelar reactions with the interstellar material and Coulomb interactions including ionization energy loss. Additional processes which might affect the energy dependence of the relative composition include Fermi acceleration in collisions of the cosmic-ray nuclei with magnetic irregularities in clouds and a rigidity dependent escape from the galaxy, or storage region, if the cosmic rays are limited to some region and do not pervade all of the universe. It is also possible that cosmic rays have passed through some material before leaving the source region, where the above processes may also occur.

In an earlier paper (Fichtel and Reames, 1966),[1] the theory of energy dependent propagation of cosmic rays through interstellar space was developed under the following assumptions: (a) The source energy/nucleon spectra of all multiply charged nuclei have the same shape, at least above 100 MeV/nucleon. (Note that, since all of the multiply charged nuclei that will be of interest at the source have nearly the same charge to mass ratio, this effectively permits the spectra to be both velocity and rigidity dependent.) (b) The relative abundance of He^3 and light nuclei ($3 \leq Z \leq 5$) at the source are negligible compared to He^4 and medium nuclei respectively. (c) The average interstellar potential path length from the source to the earth is independent of the energy/nucleon of the particle. (d) The potential path length distribution was reasonably smooth and did not contain a high percentage of very long or very short paths. In this same article, alternate approaches and models suggested in the literature were also examined.

Subsequently, new measurements on the fragmentation cross sections for cosmic rays interacting with interstellar matter have been made, and, in some instances, their values are markedly different from those previously assumed. In light of this and new measurements on the relative abundances of cosmic rays, the calculations were repeated, and the results presented in a recent paper

(Reames and Fichtel, 1967)[2] together with a consideration of the problem of possible interstellar acceleration. The relative abundances deduced in that paper indicated that there seemed to be no simple way of explaining the experimental results particularly the relative abundance of the light ($3 \leq Z \leq 5$) and medium ($6 \leq Z \leq 9$) nuclei within the limits of the assumptions.

The application of the same propagation method to different types of media including one which more nearly simulated the source region was accomplished by Durgaprasad.[3] He considered an ionized medium, one consisting of a composition which approximated that of a supernovae as nearly as possible, and a partially ionized region with a small percentage of helium as well as hydrogen to better approximate interstellar space. The results of his work showed that within the range of interest the shape of the rate of energy loss curve was virtually the same in all cases, but that the magnitude of the rate of energy loss was greater for ionized regions. Also, there was no significant difference between the rate of energy loss in pure hydrogen and the combination approximating interstellar space. Thus, in this work there is no need to be concerned about the exact interstellar composition. If the cosmic rays have spent a significant percentage of the material path length in an ionized source region there is an effect, which can generally be summarized as the suppression of heavier elements at lower energies.

One approach which goes beyond the assumption of a single path length for all particles involves assuming that the cosmic rays are in equilibrium at a given point and further that there are no spatial variations or second order energy effects. It has been shown previously[2] that even with the inclusion of a Fermi acceleration effect this approach will not give satisfactory agreement with the experimental data. Cowsik et al. (1966)[4] have suggested adding a term to the equation for loss of particles at a point which is intended to represent the escape from the galaxy in an equilibrium model. Actually, an escape of this kind should be included as a boundary condition in the solution of the complete equation and not as an additional term. Once boundary conditions must be applied, an analysis involving solving a differential equation exactly or one such as that to be presented in this paper is appropriate. The mathematical results derived from introducing a term for loss at a point are given by Cowsik et al.,[4] and in Reames and Fichtel[2]. In the latter paper the predictions are shown to disagree with the experimentally observed light to medium nuclei ratio.

In comparing experimental results to theoretical predictions, the problem could be complicated by the fact that the local solar modulation is not yet known, but the general belief is that it probably depends only on the velocity and charge-to-mass ratio of the particle. Therefore, although nuclei of the same charge-to-mass ratio, but different charges will lose energy at different rates in interstellar space, the fluxes of these particles will be modulated in the same way, thereby permitting the separation of modulation effects from interstellar energy loss and fragmentation effects.

In this paper, we wish to review and extend the previous work, first by summarizing the fundamental considerations and expanding parts of the previous work to show more clearly the nature of the problem, and secondly by pursuing the implications of the results and the possible reasons suggested in the previous paper[2] for the deviation of the experimental results from those expected on the basis of the earlier calculations. Other work on the same subject will also be reviewed at the appropriate points in the discussion. The conclusions reached after an examination of various alternatives will be presented. A two source model wherein one recent local source is superimposed on a general cosmic ray flux will be given special consideration.

II. PROPAGATION THEORY

This section will be divided into three parts. The first part will be devoted to a brief summary of the problem of the effect of the travel of particles through matter and the method used to determine the relative abundances and energy spectra after travel through any amount of matter. The second part will involve a discussion of the diffusion problem, which must be treated more explicitly now in anticipation of the need to consider more complex cases than have been examined formerly.

A. Effect of Matter

The present approach to energy – dependent propagation of cosmic rays through interstellar space is based on the one described in detail previously,[1] which includes both energy loss and fragmentation. The fundamental transport equation used there is:

$$\frac{d}{dx}\,[w_i(E_s, X)\, j_i\,(E_s,x)] + w_i(E_s,X) j_i(E_s,x)/\Lambda_i$$

$$= w_i(E_s,X) \sum_{k>i} j_k(E_s,x)/\Lambda_{ki}\,(E_s,x) \qquad (1)$$

where $j_i(E_s, x)$ is the flux per unit energy/nucleon of i-type particles of energy/nucleon E after propagation through x g/cm^2 of material given their initial energy/nucleon to be E_s, $w_i(E_s, x) = (dE/dx)_i$ for these particles, Λ_i is the loss mean free path, Λ_{ki} is the mean free path for production of i-type particles from k-type particles, and E_s is the energy/nucleon at the source. The specific assumptions mentioned in the introduction were discussed in the earlier article[1] in detail. Since the reasons for believing the first two have not changed, these assumptions will be kept; however, the third assumption will be examined later, and the fourth in sections IIB and III.

The calculations are made in steps of 0.02 g/cm^2 and the individual elements from Helium through Oxygen and the charge groups $9 \leq Z \leq 19$ and $Z > 20$ are considered. The dominant cross sections used in the fragmentation process are shown in Table I. Most of the cross-sections listed are taken from experimental measurements[5-9]. Note that the low loss cross section at low energies for VH nuclei reflects the fact that most of the total reaction cross section results in the production of nuclides of only slightly lower change; the probability of producing a nucleus with $3 \leq Z \leq 20$ for a nucleus with $Z = 26$ is extremely small in this region. Loss cross sections for H nuclei are similarly effected. Since oxygen represents the heaviest single element considered, its loss cross section is more nearly the measured total interaction cross section as in the case for lighter elements. These cross sections depend approximately upon $A^{2/3}$. The cross sections for production of light nuclei shown in Table I are now reasonably well known.[5-8]

B. Diffusion, Source Distribution, and Boundary Conditions

Previously Fichtel and Reames [1] noted that because of the smooth variation in the flux ratios from 1 to 6 g/cm^2, the assumption that all cosmic rays have traversed a given amount of material, X_0, will give essentially equivalent results to assuming a reasonable path length distribution which has an average value of X_0. That this statement is basically correct is evident also from the results of Balasubramahyan, et al.;[10] however, we now wish to consider at least some cases where there is a substantial probability for very long and, more importantly, very short path lengths. Therefore, it is appropriate at this point to review the problem of the diffusion of cosmic rays.

The flux F_i of particles of energy between E_0 and $E_0 + \Delta E$ at the observing point, r_0, at τ is given by the expression

$$F_i\,(E_0,\ r_0) = \int_{V_s} d^3r_s \int_{\tau=-\infty}^{t} d\tau \sum_{j \geq i} Y_{ji}(E_s, r_s, \tau, E_0, r_0, t)\, S_j\,(E_s, r_s, \tau) \tag{2}$$

where the S_j are the actual sources intensities,

$$\sum_{j \geq i} Y_{ji}$$

Table I
Dominant Fragmentation Cross Section (mb) vs Energy (MeV/nucleon)

E	30	50	70	100	150	200	300	500	800	1000	4000	10,000
Reaction	*											
VH loss	0	0	0	0	0	0	0	7	26	44	220	400
H loss	36	67	88	105	117	124	133	143	153	160	200	300
O loss	570	450	300	290	290	290	290	290	290	290	290	290
O → N	138	171	154	125	98	90	84	84	86	89	90	90
O → C	136	96	75	61	55	52	51	50	50	50	50	50
VH → B	0	0	0	0	0	0	0	2	4	6	24	32
H → B	0	0	3	7	11	13	16	19	22	24	24	24
O → B	3	25	38	32	25	25	25	25	25	25	25	25
C → B	114	128	118	98	73	63	54	50	49	49	49	49
O → Be	0	2	5	7	8	8	9	9	10	11	11	11
C → Be	5	18	14	11	10	9	9	11	12	12	12	12
VH → Li	0	0	0	0	0	0	0	1	3	5	15	22
H → Li	0	0	0	2	6	8	12	18	23	25	30	30
O → Li	0	2	8	16	22	26	30	35	36	36	36	36
C → Li	4	18	20	19	16	14	12	12	12	12	12	12
B → Li	36	35	32	30	30	30	30	30	30	30	30	30
Li loss	250	170	120	110	110	110	110	110	110	110	110	110

*A zero implies only that the cross section is less than 1 mb and hence negligible for the calculation.

is the propagation function for the i'th species of energy E_0 at r_0 and t arising from energy E_s at the source r_s at time τ before t. If the medium in which the particles are diffusing is isotropic with a constant density δ, or can at least be approximated by an average density, the quantity $t - \tau$ is given by the equation

$$(t - \tau) = \ell/c\beta = x/\delta c\beta \tag{3}$$

where ℓ is the path length in conventional units and x is the path length in density · length (e.g. g/cm^2). Further, if S_j is independent of time, or at least on the average over the time scales of interest, then eq. (2) becomes

$$F_i(E_0, r_0) = \int_{V_s} d^3r_s \int_0^\infty d\ell \sum_{j\geq i} Y_{ji}(E_s, r_s, E_0, r_0, \ell)\, S_j(E_s, r_s). \tag{4}$$

In the last equation, Y_{ji} includes both the effects of fragmentation and energy loss, discussed in the previous section, and the diffusion function. These two effects are strictly separable if the diffusion function is not a function of the type of particle or its rigidity. In fact, it is still a fair approximation to consider them as separable as long as there is not an appreciable change in the diffusion characteristics over the range of rigidities involved in the transit of a given particle from the source to the earth. Under the assumption mentioned, Y_{ji} is the product of a diffusion propagation term $D(\ell, E_0, r_s, r_0)$ and a material propagation term $M_{ji}(E_s, E_0, \ell)$.

Equation (4) then becomes

$$F_i(E_0, r_0) = \int_{V_s} d^3r_s \int_0^\infty d\ell\, D(\ell, E_0, r_s, r_0) \sum_{j\geq i} M_{ji}(E_s, E_0, \ell) S_j(E_s, r_s) \tag{5}$$

The last term in eq. (5) represents the result of propagation through ℓ g/cm^2 of material and is, therefore, the solution of eq. (1) described in part IIA, i.e.

$$j_i(E_0,\ell) = \sum_{j \geq i} M_{ji}(E_s,E_0,\ell)\, S_j(E_s,r_s) \tag{6}$$

Eq. (5) may now be written as

$$F_i(E_0,r_0) = \int_0^\infty \left[\int_{V_s} D(\ell,E_0,r_s,r_0)\, d^3r_s \right] j_i(E_0,\ell)\, d\ell \tag{7}$$

D is determined in the manner to be described now, and the integral over $d\ell$ is accomplished by a means of a computer in steps of 0.02 g/cm^2.

Eq. (7), in effect, simply states that the energy spectrum deduced for each type of particle for a given path length ℓ is multiplied by the relative probability of occurence of that path length, and then this product is integrated over all possible values of the path length.

In order to obtain an explicit expression for $D\, d^3r$, which is in effect a potential path length distribution for cosmic rays, specific models for the origin of cosmic rays must be considered. At one extreme is a single source region, perhaps the galactic center, and at the other perhaps is the concept of cosmic rays spread uniformly throughout the entire galaxy or even the universe and fed by sources spread throughout this whole region. An intermediate picture[11], which is often thought to be closer to the real situation, is one in which the cosmic rays observed in our galaxy are the result of many supernovae which are randomly spaced in time about a hundred years apart and probably concentrated towards the central part of the galaxy and in the spiral arms. In the first part of the work that follows we shall assume that in the past, or at least that part in which a large part of the cosmic rays observed are produced, the probability of cosmic ray production was independent of time, because of the time scales involved. This assumption implies that all of the situations which we are describing are equilibrium ones, which is reasonable for the cosmic ray case. An alternative concept is that they were all produced at one point in the past presumably the origin of the galaxy. The latter theory has the difficulty that the density of matter is such that the composition of the cosmic rays would probably be very different from that observed due to the very long path length.

Another possibility is that a single relatively close supernovae produced a significant fraction of the cosmic rays. This assumption will be explored in part IV, and the results must be reconciled with the considerable evidence suggesting the constancy, to within a factor of two of cosmic rays over time periods which are long compared to the time needed to traverse the average path length.

A few mathematical models related to the problem under consideration will now be examined and general conclusions will then be drawn from the results of the calculations. In looking at the problem of diffusion in the galaxy, we are faced with the difficulty of having little idea of an appropriate model. Therefore, we shall examine several extreme cases to try to place constraints on the type of processes which might be occurring.

The consideration will begin with the diffusion from a point source into a uniform medium of finite extent. This will be followed by the cases where the medium has an infinite extent and where there are a distribution of sources. The distribution of potential path lengths from a point source to an observation point a distance r away can be obtained readily from diffusion theory[12], and is given by the expression

$$D\Delta\ell = \frac{N}{2d^3}\,\frac{d}{r}\sum_{h=1}^{\infty} n \sin\left(\frac{n\pi r}{d}\right) \exp\left(\frac{-n^2\pi^2\lambda\ell}{3d^2}\right) \Delta\ell \tag{8}$$

where d is the distance to the edge of the medium, λ is the mean free path between scatters, ℓ is the path length, and N is a normalizing constant which depends on the intensity of the source. To obtain the expression in eq. (8), the flux at the boundary is actually assumed to be zero. When this approximation is not made a much more complex form is obtained; however, for the purposes of this work, the expression above is quite adequate. This formulation also assumes that the source has been uniform in intensity throughout the past. The calculation which follows immediately now can therefore be applied to the cosmic ray case if the time between outbursts in a region to be considered is such that $v\Delta t$ ($= \Delta\ell$) is small compared to the average distance. Taking the particular example of supernovae in our galaxy $\Delta t \simeq 100$ years for the galaxy as a whole, or from 10 to 10^2 times this value for a smaller region. If $v = c$, and there is 1 atom/cm^3, $\Delta\ell = 2 \times 10^{-2}$ to 2×10^{-3} g/cm^2 compared to an estimated average path length for cosmic rays of the order of 4 g/cm^2. The value of $\bar{\ell}$ depends somewhat on the specific model, but this value is quite adequate for this present discussion.

Notice first that as ℓ becomes sufficiently large so that $(\pi^2\lambda\ell/3d^2) >> 1$ eq. (8) becomes:

$$D\Delta\ell \xrightarrow[\ell\to\infty]{} \frac{N}{2d^3}\ \frac{d}{r}\sin\left(\frac{\pi r}{d}\right)\exp\left(\frac{-\pi^2\lambda\ell}{3d^2}\right)\Delta\ell \tag{9}$$

Any distribution considered then in a bounded medium for sufficiently large ℓ will ultimately approach an exponential character. For the cosmic ray case the condition $(\pi^2\lambda\ell/3d^2) >> 1$ for most of the range of ℓ values of interest would imply the following: From the experimental data on the light to medium ratio we have already mentioned that $\bar{\ell}$ is about 4 g/cm^2 or 2.5×10^{24} cm under the assumption of 1 atom/cc. The radius of the galaxy is about 4×10^{22} cm, which is the largest trapping region that seems reasonable to consider. Thus, λ must be much larger than 2×10^{20} cm if eq. (8) were to be a reasonable approximation over most values of ℓ. Since this would imply λ was only one or two orders of magnitude smaller than the galaxy and of the order of the thickness of the galactic plane, it seems unlikely that the equation $(\pi^2\lambda\ell/3d^2) >> 1$ holds for ℓ values of interest.

Returning to eq. (8) if $d >> r$ and $d^2 >> \lambda\ell$, the sum can be replaced by an integral. Upon completing the integral the following equation is obtained:

$$D\Delta\ell \xrightarrow[d\to\infty]{} \frac{3^{3/2}N}{4\pi^{3/2}\lambda^{3/2}\ell^{3/2}}\ \exp\left(\frac{-3r^2}{4\lambda\ell}\right)\Delta\ell \tag{10}$$

Since this equation is increasingly close to being exact as $d \to \infty$, it is the expression for an infinite isotropic medium.

The second condition probably holds for the galaxy for ℓ values of interest and the first will hold for sources which are not too distant. If the first condition does not hold very well, the physical result is not expected to be too different. Notice also that as r becomes large D_1 decreases very quickly as the square of r^2 in the exponential, except for values of ℓ much greater than those of interest. As will be seen except for a very unusual source distribution the case of large r, where $r \to d$, will not be of interest.

Consider now the case where eq.(10) is valid, but instead of one source there is a distribution of sources, with a density given by ρ, where ρ includes both the effect of varying source strength and source density. Here, we are assuming that there is no source which has been so close to the observer or so strong as to cause it to stand out over the others. If this assumption is true, the sources can adequately be represented by a distribution function ρ. If ρ is normalized such that $\iiint \rho dV = 1$, then the distribution of path lengths will be given by the expression

$$\int D\Delta\ell \, d^3r_s = \int_0^\infty \int_0^\pi \int_0^{2\pi} \frac{3^{3/2} N \, \Delta\ell}{8\pi^{3/2}\lambda^{3/2}\ell^{3/2}} \, \rho \exp\left(\frac{-3r^2}{4\lambda\ell}\right) r^2 \sin\theta d\theta d\Phi \qquad (11)$$

Letting

$$z = r\Big/\sqrt{4\lambda\ell/3}, \qquad (12)$$

yields

$$\int D\Delta\ell\, d^3 r_s = \frac{N\Delta\ell}{\pi^{3/2}} \int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} \rho z^2 \exp(-z^2)\, dz\, \sin\theta d\theta d\Phi. \tag{13}$$

Let us now consider certain special cases for the function ρ, assuming for the present that λ is not a function of position.

Case (a): Suppose the sources are uniformly distributed over all space with $\rho = \rho_0$. Then (12) becomes:

$$\begin{aligned} \int D d^3 r_s \Delta\ell &= \frac{N\Delta\ell\rho_0}{\pi^{3/2}} 4\pi \int_0^{\infty} z^2 \exp(-z^2)\, dz \\ \int D d^3 r_s \Delta\ell &= (N\rho_0)\Delta\ell\,. \end{aligned} \tag{14}$$

Notice that this result does not depend on ℓ and that it is one example wherein there are no spatial or energy gradients of second order so that when it is inserted in eq. (7) the result is identical to the approximate equilibrium at a point solution discussed earlier [2, 4].

Case (b): Suppose next that the sources are limited to some finite volume, and, to obtain an explicit expression, assume that $\rho = \rho_0$ for $r \le r_a$ and $\rho = 0$ for $r > r_a$. Then eq. (12) becomes

$$\int D d^3 r_s \Delta\ell = \frac{N\Delta\ell\rho_0}{\pi^{3/2}} 4\pi \int_0^{a} z^2 \exp(-z^2)\, dz \tag{15}$$

where

$$a = r \Big/ \sqrt{4\lambda\ell/3}\,.$$

This last integral can be integrated by parts to obtain

$$\int D\, d^3 r_s \Delta\ell = N\rho_0 \Delta\ell \left[-\frac{2a}{\sqrt{\pi}} e^{-a^2} + \frac{1}{\sqrt{\pi}} \int_0^a e^{-z^2}\, dz \right].$$

The function given in eq. (15) is plotted in fig. 1, using the tables given by Jahnke and Emde[13] to evaluate the integral.

There are now four specific distributions that have been developed, and the implication of these results will be examined since most other possibilities will lie in between these cases. For example if the distribution of sources in the medium is not centered about the observing point, the potential path length distribution will appear somewhat like a superposition eqs. (10) and (15). If the sources all lie appreciably away from the observing point the potential path length distribution will appear similar to eq. (10) only flatter and broader.

Thus far, λ has not been investigated in detail. In principal λ can be a function of both the position in space and the rigidity of the particle. In fine structure, λ will certainly vary strongly with position; whether or not it does on a larger scale is not an easy question to answer, except that it probably increases toward the outer boundary of the galaxy. Eq. (8) shows that this effect would increase the tendency towards an exponential distribution in ℓ at a smaller

ℓ value. Also, if it is assumed λ is equal to $\lambda_0(r/r_0)^\beta$ eq. (10) is replaced by

$$\frac{N'}{(r/r_0)^\beta (\lambda_0\ell)^{(3-\beta)\,(2-\beta)}} \exp\left\{-\frac{3r_0^\beta r^{2-\beta}}{(2-\beta)^2\lambda_0\ell}\right\}. \qquad (16)$$

Further considerations of this type will not be pursued because as we shall see later, the resulting predictions for cosmic ray spectra and abundances are very insensitive to changes in forms of this type.

A much more significant consideration is the possible variation of λ with rigidity. Clearly it is unlikely that all cosmic ray particles will be affected the same way by the magnetic irregularities in space. In general, a distribution in scale sizes of the irregularities would be expected, and, since the particles of lower rigidity would then be expected to encounter more significant deflections, their mean free path would be longer. However, as Parker[12] has pointed out in relation to solar particles, it is also possible for low rigidity particles to follow field lines along a large kink which would affect a high rigidity particle. Since essentially nothing is known about the dependence of mean free path on rigidity except that once the rigidity is large enough, the path length will decrease with energy, a trial function will be used with the aim of seeing what effect it has. Assume that the mean free path was given by the expression $\lambda = \lambda_0 + AR$, which states that λ approaches a constant at low rigidities and then increases in proportion to the rigidity at large values. The applications of the mathematical expression for λ is restricted by the limitation mentioned earlier in the section that D cannot depend strongly on E_0, or it is not valid to separate it from the material propagation term. (See the discussion following eq. (4).)

Notice, however, that the eq. (14) for sources spread uniformly throughout space does not depend on λ and hence cannot depend on rigidity.

III. EQUILIBRIUM MODELS

A. Comparison to Experimental Results

In this section, the resulting relative abundances of the cosmic ray nuclei will be studied as a function of the source spectral shape and the potential path length distributions developed in section II for cases of equilibrium. This study will be supplemented by a discussion of other equilibrium models suggested in the literature.

As in the earlier paper,[1] we shall choose three trial source energy spectra ranging in steepness from a power law in total energy to a power law in kinetic energy. The specific expressions used are

$$dJ/dW = C_a W^{-2.5} \tag{a}$$

$$dJ/dR = C_b R^{-2.5} \tag{b}$$

$$dJ/dE = C_c E^{-2.5} \tag{c}$$

where W is the total energy, R is the rigidity, and E is the kinetic energy per nucleon.

For comparing the results here, the light to medium nuclei, the helium to medium nuclei and the heavy to medium nuclei ratios were chosen to be displayed because the first shows some of the more significant effects since the light nuclei are secondaries and since the latter two ratios are typical of many of the others.

For comparison, the results for the case where the potential path length was a delta function, i.e. the common fixed path length approximation, are shown first in figs. 2, 3, and 4. Here the path length was adjusted to give a high-energy L/M value consistent with the data. The path length distribution will in general be normalized in this way whenever there is a free parameter. Notice first that the better cross-section data now available no longer indicate that any increase in the light to medium nuclei ratio should be seen at energies in the neighborhood of 200 to 500 MeV. Therefore, if the increase seen by some experimenters is real, it is not a fragmentation effect. This effect does not depend on the model as we shall see. Notice also that L/M is essentially independent of energy and that He/M rises very rapidly at low energies for steep spectra.

Turning now to the case of a point source expanding into an infinite isotopic medium, eq. (10) applies, and the results of the calculation outlined in the previous section lead to figs. 5, 6, and 7. These graphs show that there is little difference from the previous case as expected since eq. (10) states that there is not a heavy weighting of very short nor very long path length.

The next case considered in section IIB was the uniform distribution of sources over an infinite medium, which led to a potential path length independent of ℓ given by eq. (12a). Here, unlike the other cases, there is no adjustable parameter which can be used to normalize the result to the high energy L/M value. Hence, since the calculated light to medium nuclei value shown in fig. 8 disagrees with the observed one at high energies, this case can be rejected at once, when λ is a constant.

An important more general statement can now also be made. It was pointed out in section IIB, that the same approach to a solution, namely simply integrating over all path lengths with equal weight, also applied to any equilibrium situation wherein there are not serious gradients in space or energy. Hence, the experimental results on the light to medium nuclei ratio, also exclude this class of model. One important set in this class is any universal model for the cosmic rays except those which somehow would predict substantial spatial or energy gradients.

Eq. (15) referred to sources limited to a region around the observing point. Figs. 9 and 10 shows that the light to medium nuclei ratio decreases markedly at low energies whereas the helium to medium nuclei ratio remains fairly constant. These characteristics appear in cases where there is a reasonably high probability for short path lengths. The reason is that the low energy spectra changes rapidly with distance traversed because the rate of energy loss is an increasing function of decreasing energy. The result is that the steep initial spectrum is bent downward quickly; so, if there is a relatively high probability for short path lengths, the low energy region reflects the source characteristics quite strongly. Thus, a relative low abundance of light nuclei is seen, and the helium tomedium nuclei ratio has not had a chance to increase much as it will with additional passage through matter because of the greater rate of energy loss of heavier nuclei.

Another case of possible interest is the one in which all of the sources are at a distance which is greater than some value. This possibility can be approximated by assuming that sources are uniformly distributed beyond some radius r_0 from the observing point. The resulting path length distribution would then be obtained by subtracting some multiple of eq. (15) from eq. (14). Regardless of the assumptions about the value of λ, except that it is a constant, the L/M value will now be even larger at high energies than for the case of uniformly distributed sources since the average path length is even larger. Thus, this model by itself is not in agreement with the experimental result.

The results of the above show that none of the above distribution give a very satisfactory agreement with experimental results on the light and medium nuclei unless the increase at low energies due to the modulation effect were much stronger than expected.

Considering next the case where λ is possibly a function of rigidity. Fig. 11 shows the result for a point source expanding into an infinite isotopic medium with $\lambda = a(1 + bR)$, i.e. essentially a constant at low rigidity and proportional to rigidity for large values. The light to medium nuclei ratio variation is in reasonable agreement with experiments over the measured range as are the other ratios, but it should be remembered that this form would predict that the light to medium nuclei ratio would decrease at high rigidities. In general, it seems possible that λ should ultimately increase with rigidity, and, hence, if the cosmic ray sources are in the galaxy, then the light to medium ration should decrease at very high energies. However, the variation with energy could be much smaller than that predicted by a λ dependence as extreme as the one chosen above to obtain agreement at low energies. Further, such an extreme rigidity dependence implies a very steep source spectrum, $j_s \sim R^{-1.5}$ instead of $j_s \sim R^{-2.5}$. Using an approach wherein it is assumed that the variation of the light to medium nuclei with energy is in fact due to a variation of the mean potential path length with rigidity, Apparao,[14,15] and Biswas, et al.[16,17] deduced a variation of path length with rigidity which peaks around 200 to 500 MeV/nucleon, similar to the result here. Kaplon and Skadron[11] suggested that this dependence of the path length on energy/nucleon may be the result of a rigidity dependent mechanism at the source. It could also be due to a rigidity dependent path in space as mentioned earlier. It seems difficult to explain why the rigidity dependence needed to obtain agreement with the experimentally observed light to medium nuclei ratio should exist. Specifically why should there be a strong rigidity dependence between 0.3 and 1.5 BeV/nucleon and apparently very little above, assuming the light to medium nuclei ratio is approximately constant above 1.5 BeV/nucleon. If the light to medium nuclei ratio should continue to decrease markedly with increasing energy/nucleon, then a very flat source spectrum is implied as mentioned before.

Another set of components of interest are deuterium and He^3. These elements are presumably nearly absent in the source; so whatever fluxes are seen are presumably secondary. Experimental data[18-22] on these nuclear species only exist in the low energy ($\lesssim$ 300 MeV/nucleon) region. The interpretation of the He^3 data is complicated by the fact that the He^3 charge to mass ratio is appreciably different from that of the parent nuclei, and, hence, He^3 is modulated differently from He^4 in the solar system in a way which is not yet well determined. Ramaty and Ligenfelter[23] have considered the problem in detail, actually using the combined data to estimate the solar modulation, and found the data to be consistent with an average interstellar path length of 4 ± 1 g/cm^2. Since their work is complete in itself and agrees with that of most others[17-19,24] who have considered this particular problem, the calculation related to these secondary particles will not be repeated here. Rather it will simply be pointed out that this result is consistant with the results presented already for the heavier nuclei in the low energy region except for spectra which are quite steep, and, as in the case of the heavier nuclei, the result is generally not sensitive to the particular diffusion model chosen.

A serious problem arises, however, in relation to the one measurement made so far on the relative abundance of fluorine. After the cosmic rays have passed through 3 to 5 g/cm^2 of material, the fluorine abundance should be about 1 or 2% of the abundance of oxygen or carbon due to fragmentation of elements heavier than fluorine. The appropriate cross section are not really known, but by analogy to similar reactions such as $F^{19}(p, pn)F^{18}$ a reasonable estimate can be made. The principal cross section is the one for the production of fluorine from neon, which is about 40 millibanks at 100 MeV/nucleon and decreases towards higher energies. Comstock et al.[25] found experimentally a fluorine to oxygen ratio of $\lesssim 1.4 \times 10^{-3}$ for an energy interval around 10^2 MeV/nucleon. Comstock et al.[25] and Burbidge et al.[26] both suggested a two component model to explain this result, one component of the cosmic rays having gone through 0 g/cm^2 and the other through 20 g/cm^2. These two components are then added in such a way to give the correct light to medium ratio. Using the procedure outline in section IIA, the composition at 20 g/cm^2 was calculated and added to the composition at 0 g/cm^2 in proportions which gave the light to medium ratio observed experimentally at 100 MeV. For an initial spectra wherein $J \sim R^{-1.5}$ a fluorine to oxygen ratio of about .01 was obtained. Similar results were obtained for $J \sim W^{-1.5}$ and $J \sim E^{-1.5}$. Further, a lithium to boron ratio of about 5:3 is obtained compared to the experimentally measured value of about 2:3. Finally, if the two spectra are added to give the correct L/M ratio at low energies, a result inconsistent with the data is obtained at high energies because the 20 g/cm^2 spectrum is much flatter and hence strongly dominates at high energies. Thus, when examined in detail this model also seems not to lead to predictions consistent with experimental results.

Various other assumptions including those already discussed here will also not lead to a fluorine to oxygen ratio which is nearly as low as the experimental result of Comstock et al.[25] Whether the fragmentation occurred in the source or in the interstellar medium is also irrelevant. There are still two possible ways in which the result could be consistent with the fragmentation concept of the formation of light nuclei. Firstly, the assumed cross sections could be wrong; secondly, the experimented measurement of the upper limit of fluorine could be wrong. Therefore, it would be extremely desirable to measure the neon to fluorine cross section as well as the production of fluorine by heavier elements. Also in view of the very unique character of the fluorine abundance measurement, confirmation by a second experiment would be desirable.

B. Discussion of the Apparent Failure of the Equilibrium Model

The failure to achieve agreement between the prediction of the models described thus far in this work and experimental results may possibly result from several features. In the following paragraphs we shall examine what to us appear to be the most obvious potential difficulties with the present approach and hence possibilities for the discrepancies, but first it is worth reemphasizing that the alternatives are being considered below because there no longer seems to be any way to resolve the dilemma within the framework already discussed.

(A) Propagation through material in interstellar space or in the source may not be the dominant process for producing light nuclei and/or they maybe emitted directly by the source. This possibility has been considered from time to time,[27,1] but, to our knowledge, no formal treatment in the literature exists. There is no means known to us of forming the nuclei such as Li, Be, B, He^3, F during the acceleration process except by fragmentation. Also, it is not yet clear how the rather unique variation of the light to medium nuclei ratio with energy observed experimentally could be derived theoretically.

(B) There may be further inadequacies in the cross section data. This possibility, although once of great concern, no longer seems as significant, due to the extensive new data on most of the significant cross sections. There are still some important gaps in the data, such as the production of fluorine.

(C) The source spectra for particles of the same charge to mass ratio may not be the same. The theoretical reasons for believing similar source spectra exist at the source were outlined previously.[1] The existing experimental evidence for helium, medium, and heavy nuclei is roughly in agreement with the assumption of similar source spectra and an average interstellar path of a few g/cm^2. This last statement is certainly true of carbon and oxygen the principal parents of the light nuclei. Finally, there is no known way, consistent with the observed experimental data, for the spectra to differ in a way which would significantly help in resolving the dilemma of the light to medium nuclei ratio.

(D) The observed features of the galaxy are considerably more complicated than those of the models. A reexamination of the models show that additional complications are not likely to resolve the problem unless they either (1) introduce a very unlikely rigidity dependence for the mean path length for scattering or (2) introduce a very unique rigidity dependent trapping at the source.

(E) The energy spectral measurements on the light or medium nuclei and other species are wrong. Although this may be a possibility due to the difficulty of charge and energy measurements in the intermediate energy range, for the purpose of this work, we shall assume they are correct until subsequent work indicates otherwise.

(F) Solar Modulation: There is almost certainly a rigidity dependence in the solar modulation, but its magnitude and form are not yet certain. Since light nuclei have a slightly higher rigidity for a given velocity, the effect of the rigidity dependent modulation is generally to increase slightly the light to medium ratio. Estimates of the degree of modulation by Ramaty and Lingenfelter[23], Gloeckler and Jokipii[28], and Durgaprasad et. al.[29] seem to indicate that the increase is at most a 2 to 6% effect around 300 to 500 MeV/nucleon which is not enough to affect the previous discussion significantly, and would, for the most part, be hidden in the experimental errors. Under extreme assumptions, e.g. assuming the modulation function as of the form exp $(-C/\beta R)$ down to low energies, the modulation effect could increase the light to medium ratio by 5 to 15% at about 100 MeV/nucleon, but more likely the rigidity dependence of the modulation decreases at low energies making the effect quite small. If, in the extreme, the modulation were that necessary to produce the low energy cosmic ray spectrum resulting from the supernovae theory proposed by Colgate and Johnson,[30] a significant portion of the difference between the observed and predicted light to medium nuclei ratio could be explained by a modulation mechanism which was rigidity dependent. However, the helium to medium ratio at low energies would then be very large and inconsistent with observations by large factors. Also, the energy density of the cosmic rays in the galaxy would then be very large.

(G) The cosmic rays come from more than one source type. Assuming more than one source type in an equilibrium model does not resolve the difficulties unless one introduces a unique composition effect at the source. The case of a non-equilibrium model will be considered in the next section.

IV. NON EQUILIBRIUM MODELS

In exploring this possibility, it must be remembered that there is some evidence from meteorites that the cosmic rays have been constant on the average within a factor of a few for a million years or more. The proposed sources must also produce the observed energy spectrum. The simplest, and in many respects the most likely, model of this type is one in which the particles from a single recent relatively close source are superimposed on the fluxes from all the other sources, whose effects can be represented reasonably well by the models discussed earlier. This possibility will be treated further in the next several paragraphs.

The potential path length distribution for particles observed from a single source will be very different from the ones already discussed. Assuming the particles are released within a short period, at least short compared to the time since their release, the path length of any observed particle will just be the velocity • time; hence, each energy interval will have a unique potential path length which will be proportional to the particle's velocity. The relative intensity will be determined from an assumed diffusion picture, the work in sections IIA and IIB, and the path length determined by the velocity, time, and interstellar density.

In allowing this new degree of freedom, we shall return to what appears to us to be the most reasonable or at least the simplest model for the remainder of the cosmic rays, namely a uniform distribution of sources in space and time diffusing into an infinite isotopic medium. This model has at least two other advantages. First, the results are independent of the diffusion coefficient. Secondly, it applies both to the cosmic ray model in which the cosmic rays pervade the whole universe in equal intensity and the one in which the diffusion in the galaxy is slow, as present models would suggest, and there is a sufficient number of galactic sources spread fairly randomly – at least in the plane. The results of the earlier work showed that there was reasonable agreement with the experimental data at low energies, but not at high energies.

If a relatively new close source has occurred, the high energy particles will be reaching us more efficiently than the lower energy ones even if the mean free path for scattering is independent of energy simply because the diffusion coefficient is proportional to β. In order to make the discussion quantitative, isotropic diffusion will be assumed and specifically it will be assumed that the diffusion picture described by eq. (10) is valid. Eq. (10) gives the relative probabilities of various path lengths as a function of ℓ and hence for β in this case by using eq. (3) since $(t - \tau)$ is a constant now. The intensity of the various components for a given source spectral shape can then be obtained. There are effectively two adjustable parameters in eq. (10), the constant multiplying $\beta^{-3/2}$ at the front and the constant multiplying β^{-1} in the exponential, since the other constants multiplying β to give ℓ can be absorbed in these two. The first of the constants was adjusted to give the observed light to medium nuclei ratio at high energies when this single source is added to the general background. At high energies this single source would represent about 40% of the total cosmic ray flux. The second constant was adjusted to give the best fit to the light to medium nuclei data. The choice of this second constant implies that the cosmic ray intensity will increase substantially in the future reaching a maximum of up to ten times the present value at high energies. Without knowing r^2/λ, it is not possible to give a precise time scale, but 10^5 to 10^6 years is probably reasonable.

The results of the addition of this source calculated in the manner just described superimposed on a flux of isotropic sources in an isotropic medium is shown in figs. 12, 13, and 14. A reasonable agreement with experimental data is possible with a source spectrum whose steepness lies between $j_s \sim W^{-2.5}$ and $j_s \sim R^{-2.5}$.

V. SUMMARY

A comparison of the experimental data to the predictions of a large number of equilibrium models for the cosmic ray propagation showed that none of the models for which the mean free path was independent of rigidity yielded results which were in agreement with the experimental data, expecially the light to medium nuclei ratio. Further, the models selected were sufficiently varied and covered enough extremes so that other likely models which come to mind would give results intermediate between those predicted by some of the cases considered here, and, therefore, would be in disagreement with the experimental data also. As has been mentioned before, agreement with the light to medium nuclei ratio can be obtained by choosing an appropriate dependence of the potential path length on energy. However, this dependence appears to be difficult to justify.

One particular experimental result, namely the fluorine to oxygen ratio at low energies, about 100 MeV/nucleon, seems to be in basic disagreement with the fragmentation concept, and, therefore this cosmic ray abundance ratio and the relevant cross sections deserve serious attention.

Several possible explanations for the difference between theoretical predictions for the relative abundances of charges and the observations were considered with a particular emphasis on the light, medium, heavy and very heavy nuclei as groups, since considerable data exists for these species. It was seen that such considerations as the light nuclei being produced in a way other than fragmentation, inadequate cross section data, different source spectra, a very complex galactic picture, poor experimental measurements, and solar modulation either did not seem to be likely explanations for the deviations on the basis of directly applicable data, or were very unlikely to provide a acceptable solution within the framework of our present understanding of other phenomena.

One possible explanation which removes the difficulties associated with the light to medium nuclei ratio and still gives reasonable agreement with the medium, heavy, and very heavy relative abundances is a non-equilibrium model, wherein there has been one recent relatively close source that is making a significant contribution to the locally observed cosmic radiation. If this concept is correct, the composition of the high energy cosmic rays when examined in detail will reflect the fact that about half of the high energy cosmic rays have gone through a relatively small amount of material. Further, it is possible that the energy spectrum of the "recent, close" source is slightly different than the average resulting in a variation in the composition at large cosmic ray energies. It would be very valuable, therefore, to have detailed composition measurements of the cosmic radiation at high energies, as well as low energies, to compare to the various possible predictions.

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FIGURE CAPTIONS

Fig. 1. The function

$$\frac{4}{\sqrt{\pi}} \int_0^a x^2 e^{-x^2} dx \text{ plotted as a function of } \frac{1}{a^2} \text{ where } \frac{1}{a^2} = \left(\frac{4\lambda}{3\, r_0^2}\right) \ell$$

Fig. 2. The light to medium nuclei ratio for a potential path length of 4.5 g/cm^2 for the source spectra indicated in the figure.

Fig. 3. The helium to medium nuclei ratio for a potential path length of 4.5 g/cm^2 for the source spectra indicated in the figure.

Fig. 4. The heavy to medium and the very heavy to medium nuclei ratios for a potential path length of 4.5 g/cm^2 for source spectra of the form $j_s = R^{-2.5}$

Fig. 5. The light to medium nuclei ratio for the case of a point source expanding into an infinite isotropic medium for the source spectra indicated in the figure.

Fig. 6. The helium to medium nuclei ratio for the case of a point source expanding into an infinite isotropic medium for the source spectra indicated in the figure.

Fig. 7. The heavy to medium nuclei ratio and the very heavy to medium nuclei ratio for the case of a point source expanding into an infinite isotropic medium for a spectrum of the form $j_s = R^{-2.5}$.

Fig. 8. The light to medium nuclei ratio for the case of a uniform distribution of sources over an infinite isotropic medium for the source spectra indicated in the figure.

Fig. 9. The light to medium nuclei ratio for the case of a uniform distribution of sources contained within a sphere of radius R_0 for the source spectra indicated in the figure.

Fig. 10. The helium to medium nuclei ratio for the case of a uniform distribution of sources contained within a sphere of radius R_0 for the source spectra indicated in the figure.

Fig. 11. The light to medium nuclei ratio for a point source expanding into an infinite isotropic medium with $\lambda = a\,(1 + bR)$ for a source spectral of the form $j_s = R^{-2.5}$. Note that fig. 5 shows that the choice of source spectra is not critical.

Fig. 12. The light to medium nuclei ratio for the case of a relatively recent local source superimposed on the flux of isotropic sources in an isotropic medium described in the text in section III for the source spectral shapes given in the figure. References for data point are given in Reames and Fichtel[2].

Fig. 13. The helium to medium nuclei ratio for the case of a relatively recent local source superimposed on the flux to isotropic sources in an isotropic medium described in the test in section III for the source spectral shapes

given in the figure. References for data points are given in Fichtel and Reames.[2]

Fig. 14. The heavy and very heavy to medium nuclei ratios for the case of a relatively recent local source superimposed on the flux of isotropic sources in an isotropic medium described in the text in section III for the source spectral shapes given in the figure. References for data points are given in Fichtel and Reames,[2] except points indicated by squares, which are from Omes and Webber.[31]

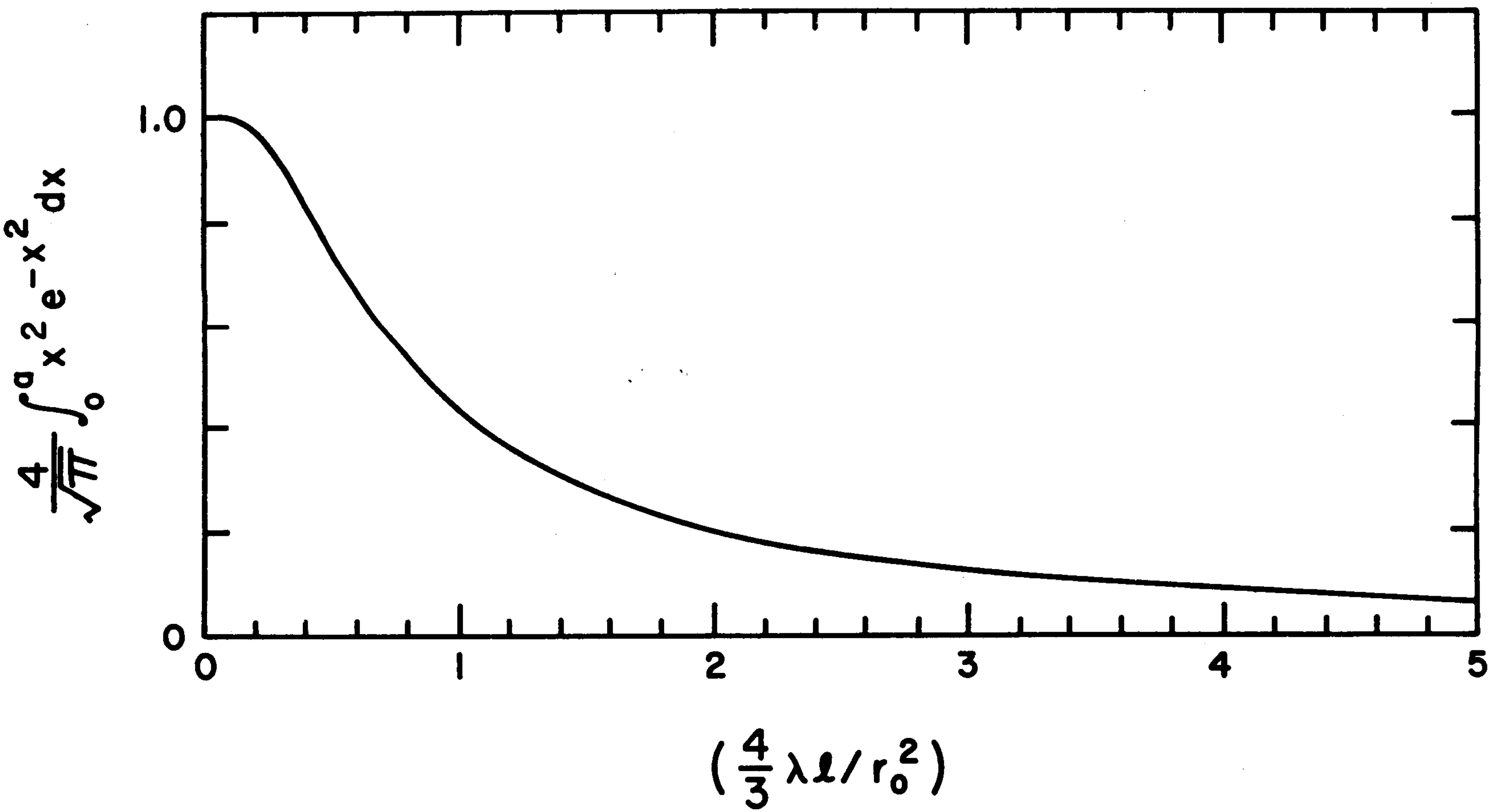
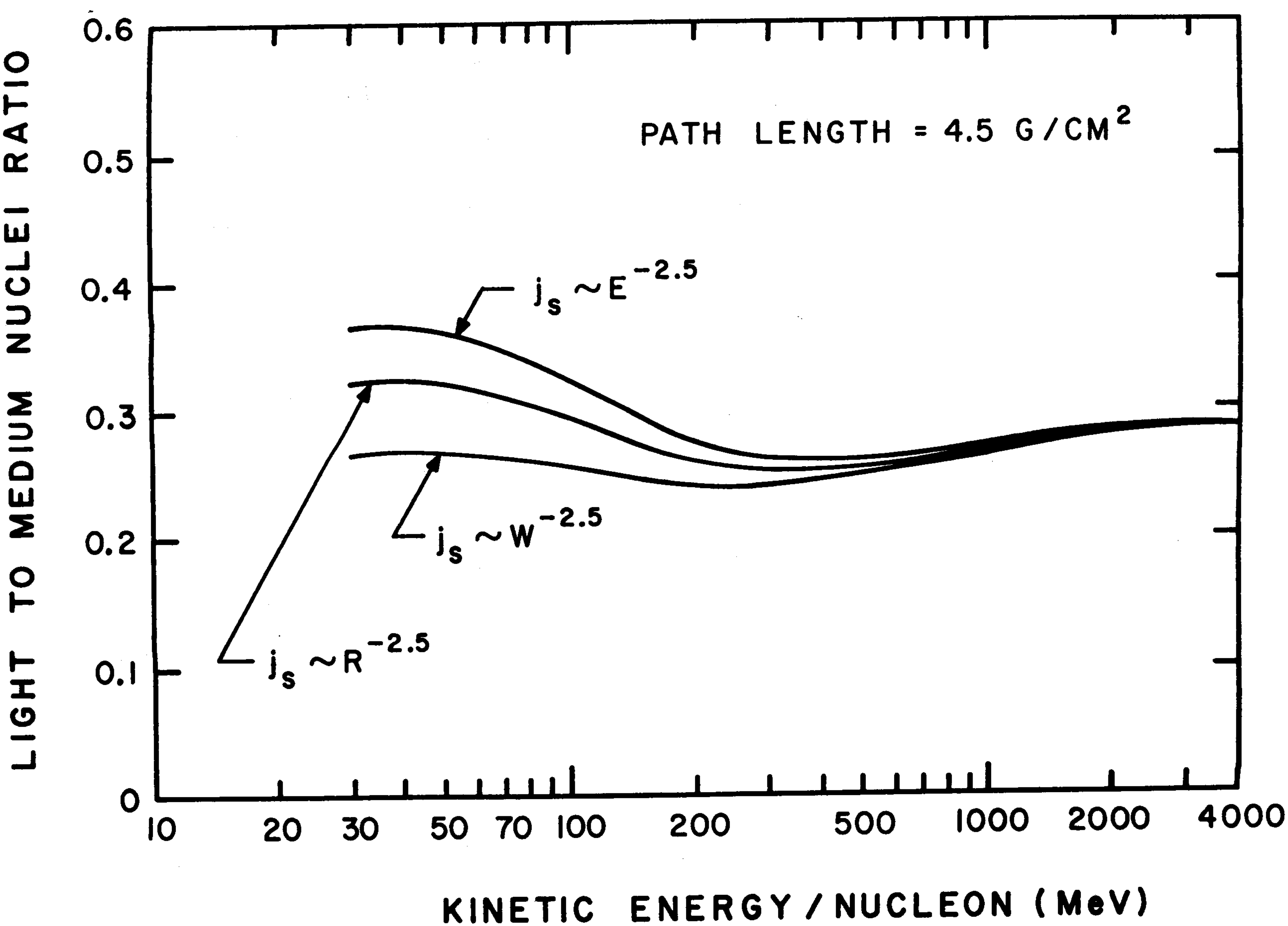
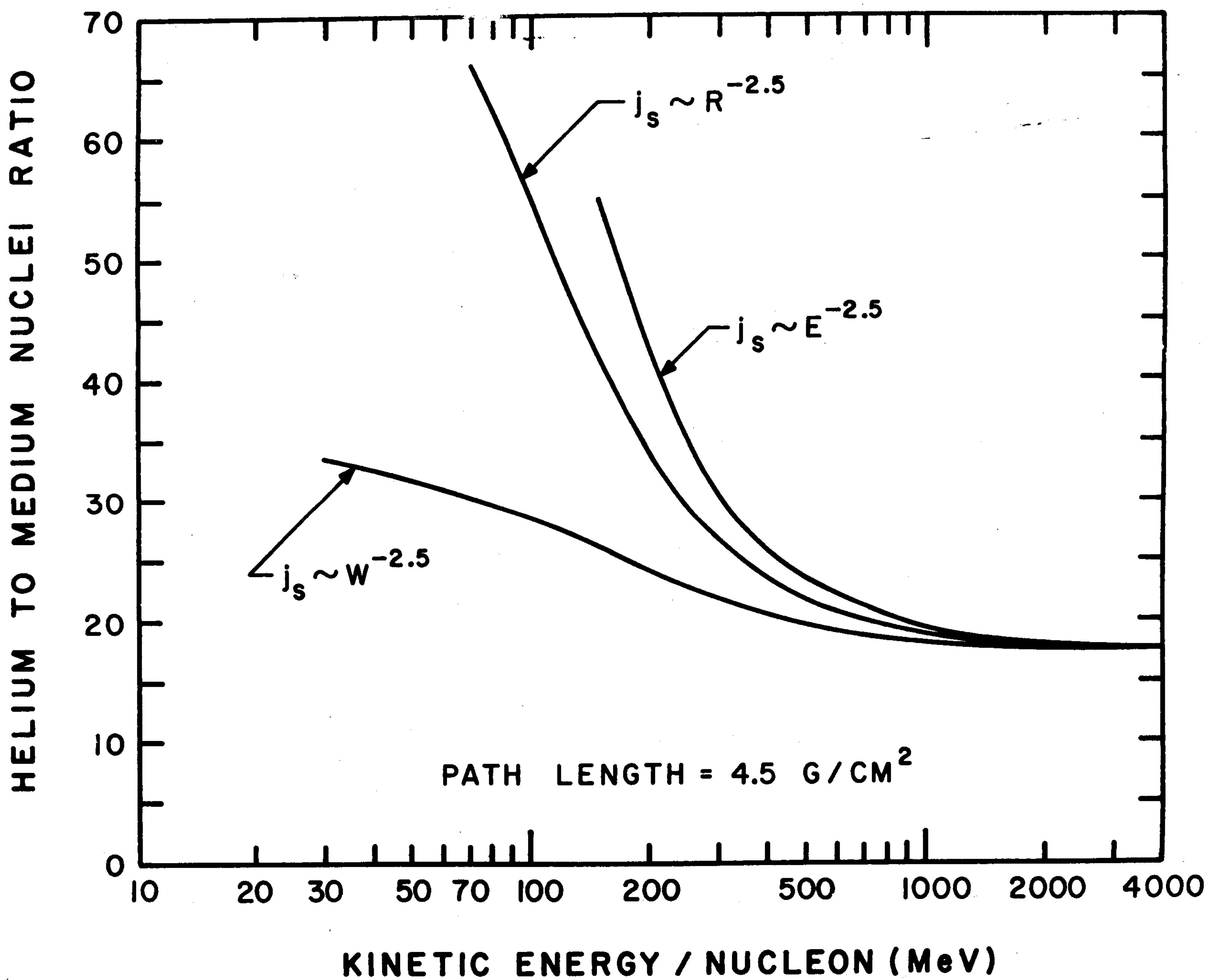

$\frac{4}{\sqrt{\pi}}\int_0^a x^2 e^{-x^2} dx$
1.0
0
0
1
2
3
4
5
$\left(\frac{4}{3}\lambda \ell / r_o^2\right)$

FIGURE 1



LIGHT TO MEDIUM NUCLEI RATIO
0.6
0.5
0.4
0.3
0.2
0.1
0
PATH LENGTH = 4.5 G/CM^2
$j_s \sim E^{-2.5}$
$j_s \sim W^{-2.5}$
$j_s \sim R^{-2.5}$
10
20
30
50
70
100
200
500
1000
2000
4000
KINETIC ENERGY / NUCLEON (MeV)

FIGURE 2



70
60
50
40
30
20
10
0
HELIUM TO MEDIUM NUCLEI RATIO
$j_s \sim R^{-2.5}$
$j_s \sim E^{-2.5}$
$j_s \sim W^{-2.5}$
PATH LENGTH = 4.5 G/CM^2
10
20
30
50
70
100
200
500
1000
2000
4000
KINETIC ENERGY / NUCLEON (MeV)

FIGURE 3

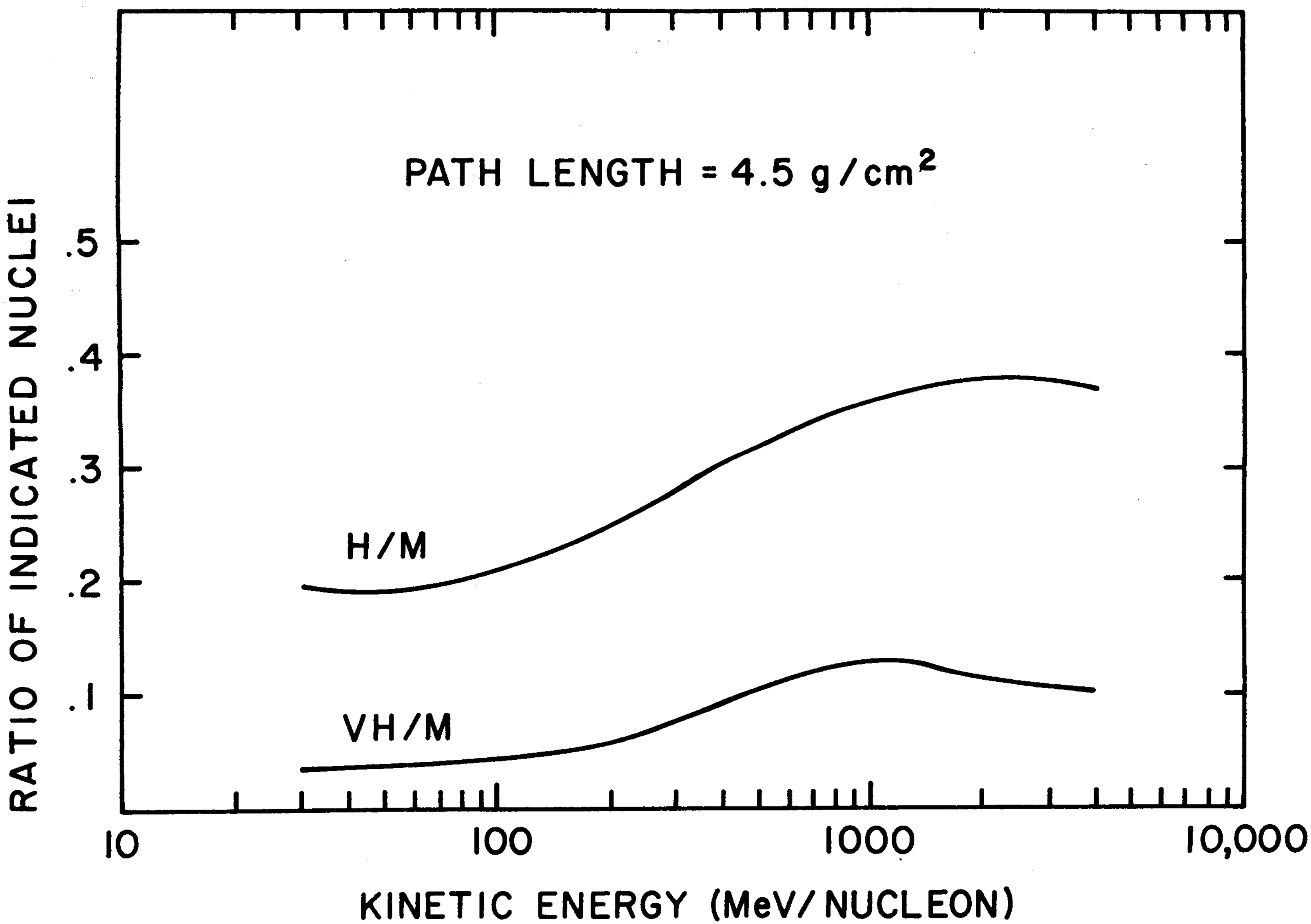
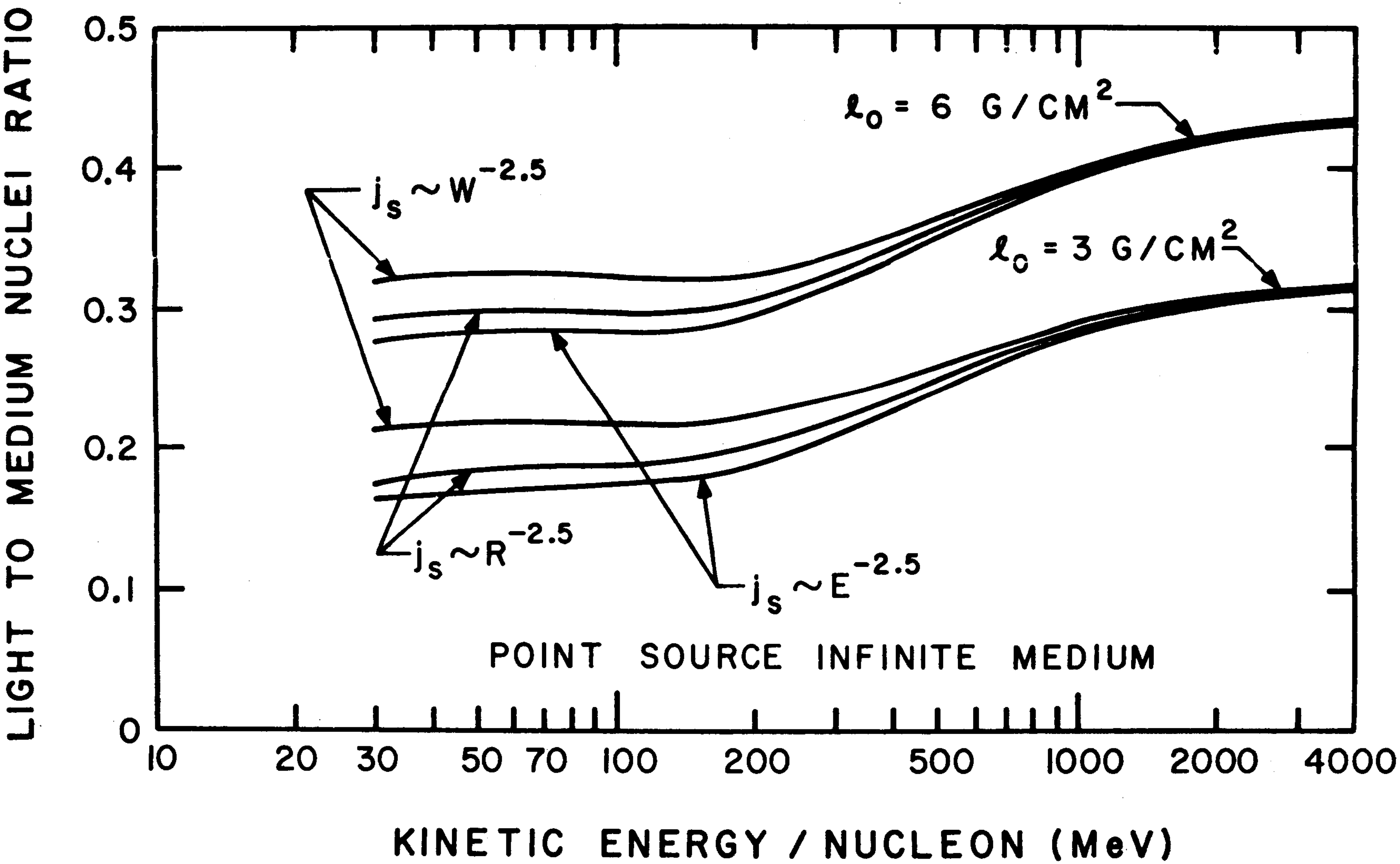
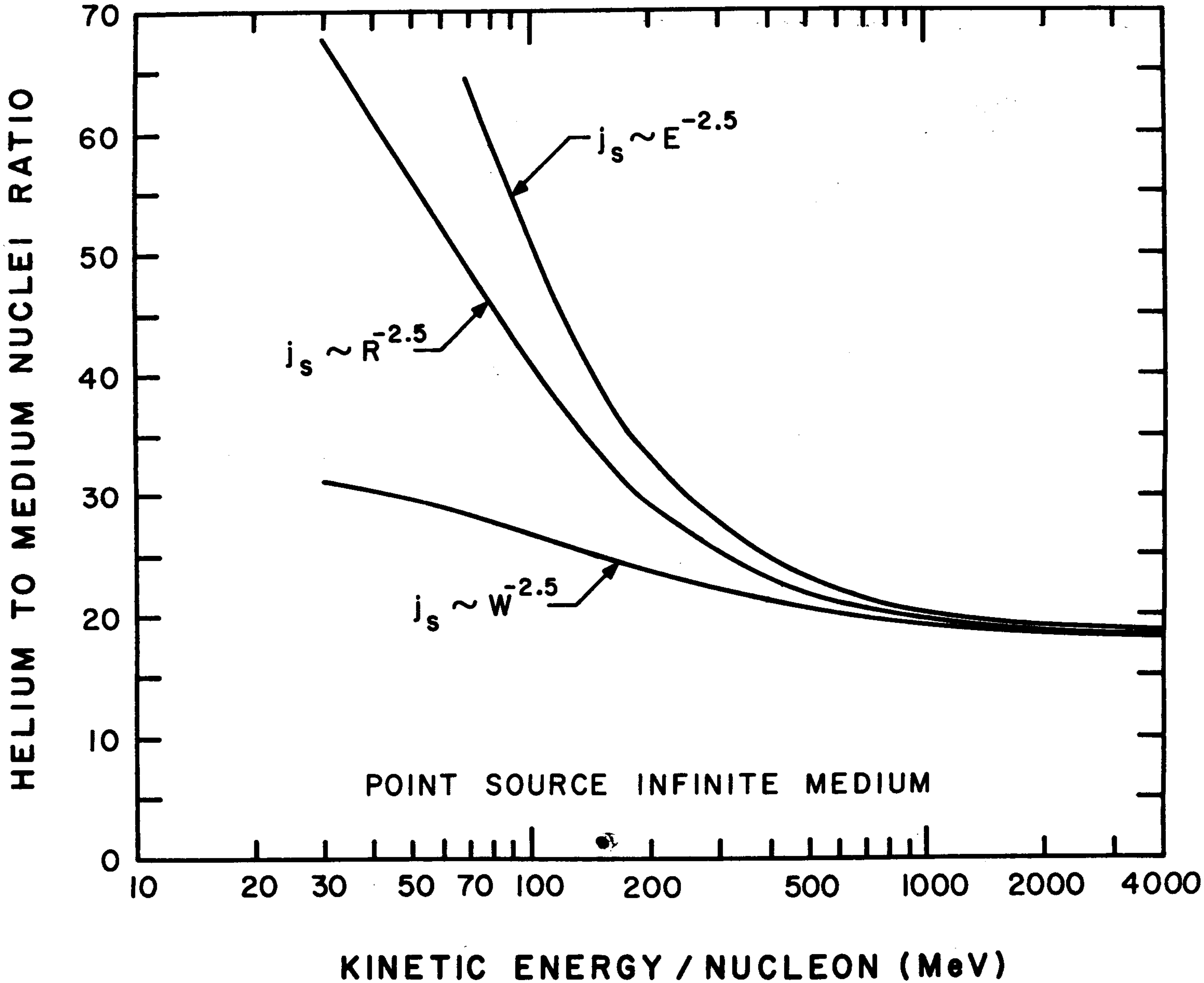

PATH LENGTH = 4.5 g/cm²
H/M
VH/M
RATIO OF INDICATED NUCLEI
.1
.2
.3
.4
.5
10
100
1000
10,000
KINETIC ENERGY (MeV/NUCLEON)

FIGURE 4



LIGHT TO MEDIUM NUCLEI RATIO
0.5
0.4
0.3
0.2
0.1
0
10
20
30
50
70
100
200
500
1000
2000
4000
$\ell_0 = 6$ G/CM^2
$\ell_0 = 3$ G/CM^2
$j_s \sim W^{-2.5}$
$j_s \sim R^{-2.5}$
$j_s \sim E^{-2.5}$
POINT SOURCE INFINITE MEDIUM
KINETIC ENERGY / NUCLEON (MeV)

FIGURE 5



HELIUM TO MEDIUM NUCLEI RATIO
70
60
50
40
30
20
10
0
10
20
30
50
70
100
200
500
1000
2000
4000
$j_s \sim E^{-2.5}$
$j_s \sim R^{-2.5}$
$j_s \sim W^{-2.5}$
POINT SOURCE INFINITE MEDIUM
KINETIC ENERGY / NUCLEON (MeV)

FIGURE 6

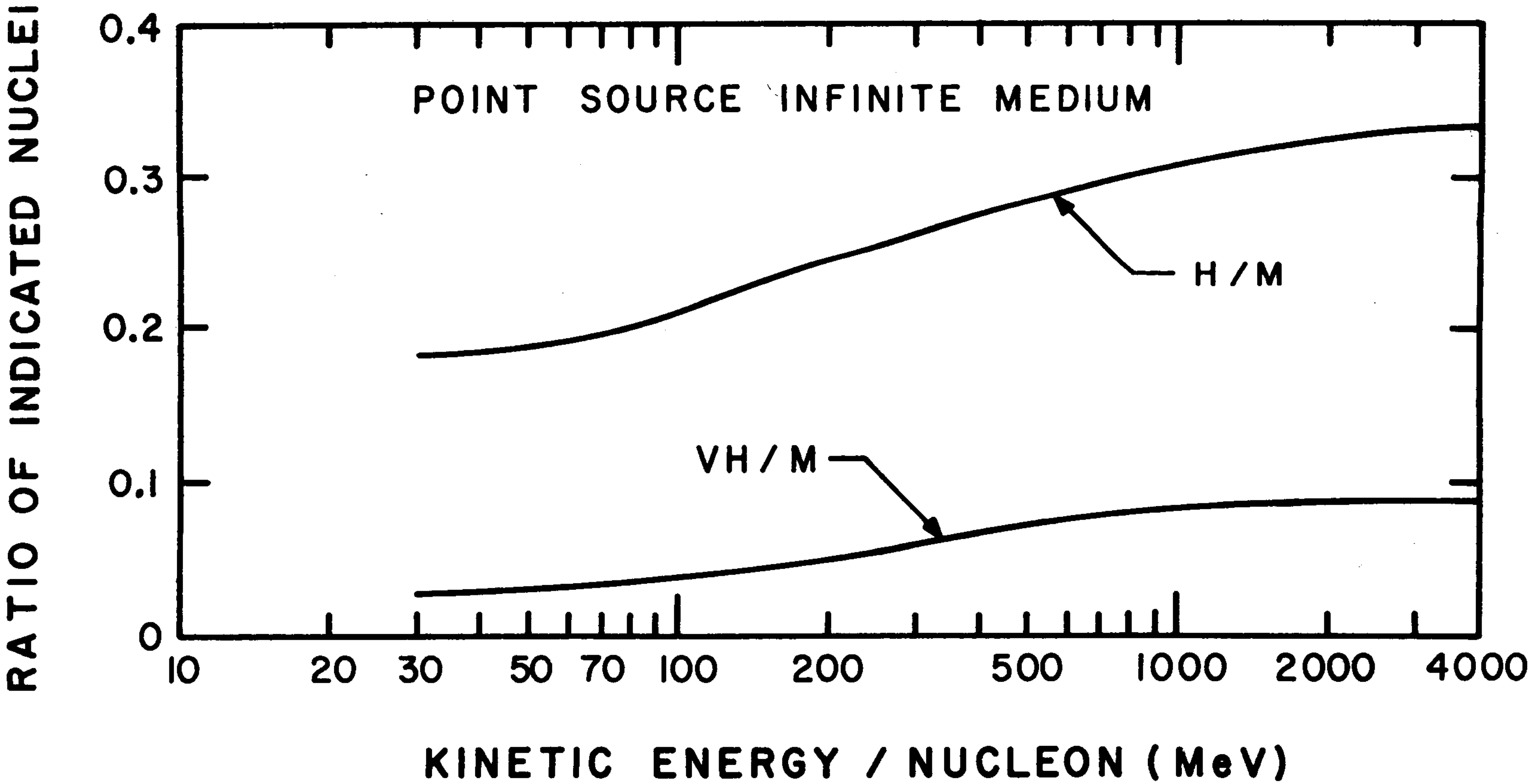

RATIO OF INDICATED NUCLEI
0.4
0.3
0.2
0.1
0
POINT SOURCE INFINITE MEDIUM
H / M
VH / M
10
20
30
50
70
100
200
500
1000
2000
4000
KINETIC ENERGY / NUCLEON (MeV)

FIGURE 7

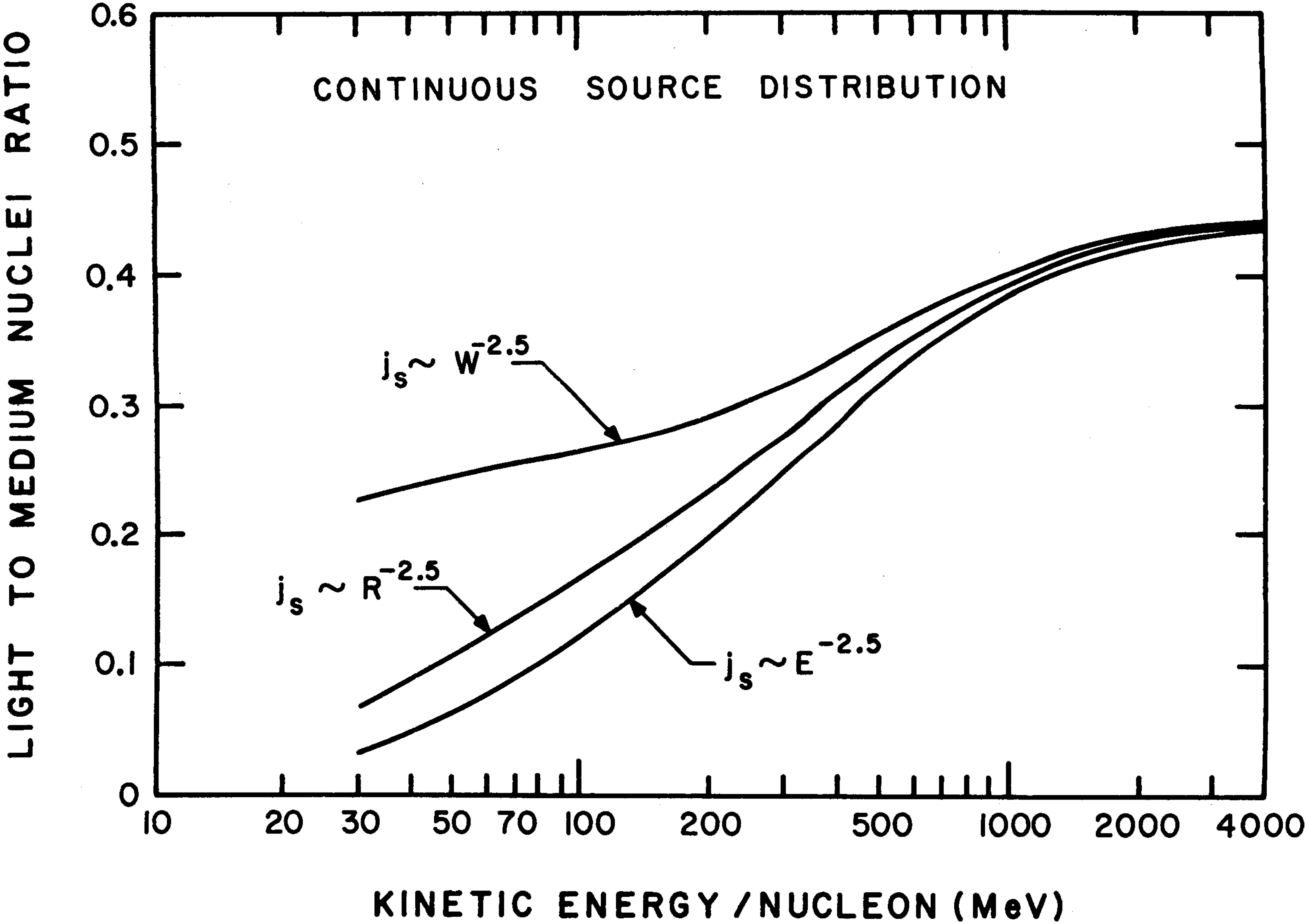

CONTINUOUS SOURCE DISTRIBUTION
$j_s \sim W^{-2.5}$
$j_s \sim R^{-2.5}$
$j_s \sim E^{-2.5}$
LIGHT TO MEDIUM NUCLEI RATIO
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
50
70
100
200
500
1000
2000
4000
KINETIC ENERGY / NUCLEON (MeV)

FIGURE 8

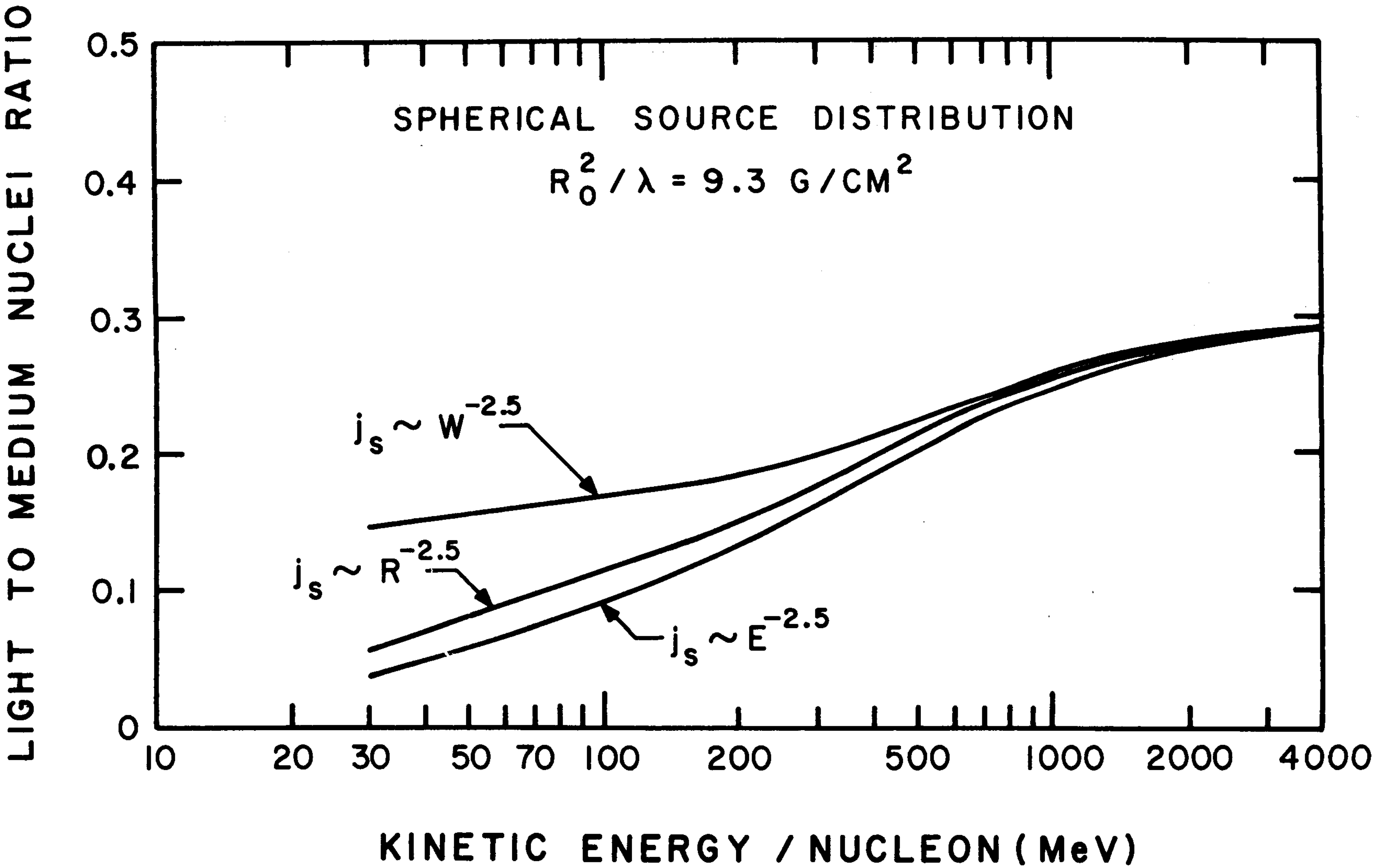

SPHERICAL SOURCE DISTRIBUTION
$R_0^2/\lambda = 9.3$ G/CM^2
$j_s \sim W^{-2.5}$
$j_s \sim R^{-2.5}$
$j_s \sim E^{-2.5}$
0.5
0.4
0.3
0.2
0.1
0
10
20
30
50
70
100
200
500
1000
2000
4000
LIGHT TO MEDIUM NUCLEI RATIO
KINETIC ENERGY / NUCLEON (MeV)

FIGURE 9

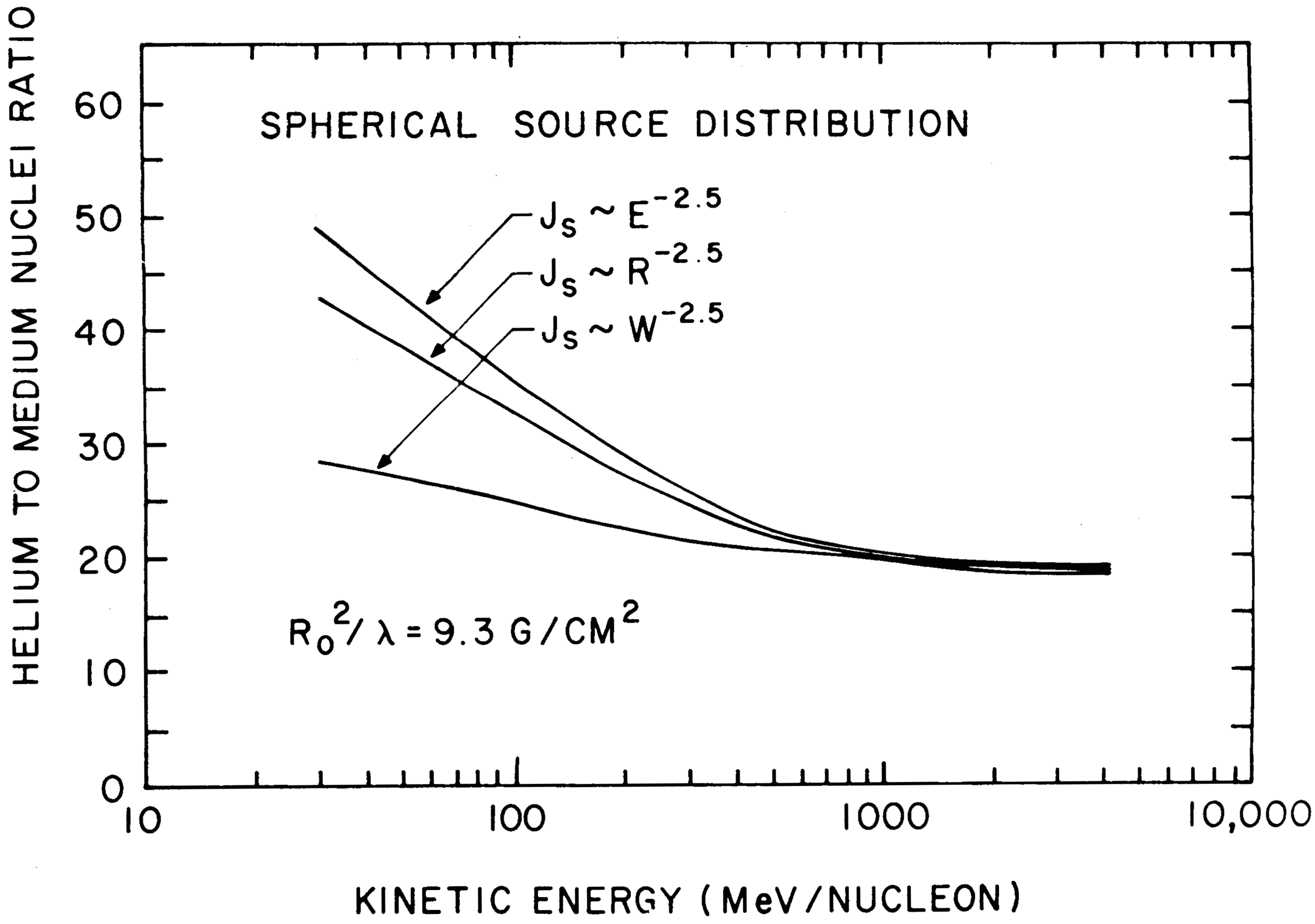

HELIUM TO MEDIUM NUCLEI RATIO
60
50
40
30
20
10
0
10
100
1000
10,000
SPHERICAL SOURCE DISTRIBUTION
$J_s \sim E^{-2.5}$
$J_s \sim R^{-2.5}$
$J_s \sim W^{-2.5}$
$R_o^2/\lambda = 9.3$ G/CM^2
KINETIC ENERGY (MeV/NUCLEON)

FIGURE 10

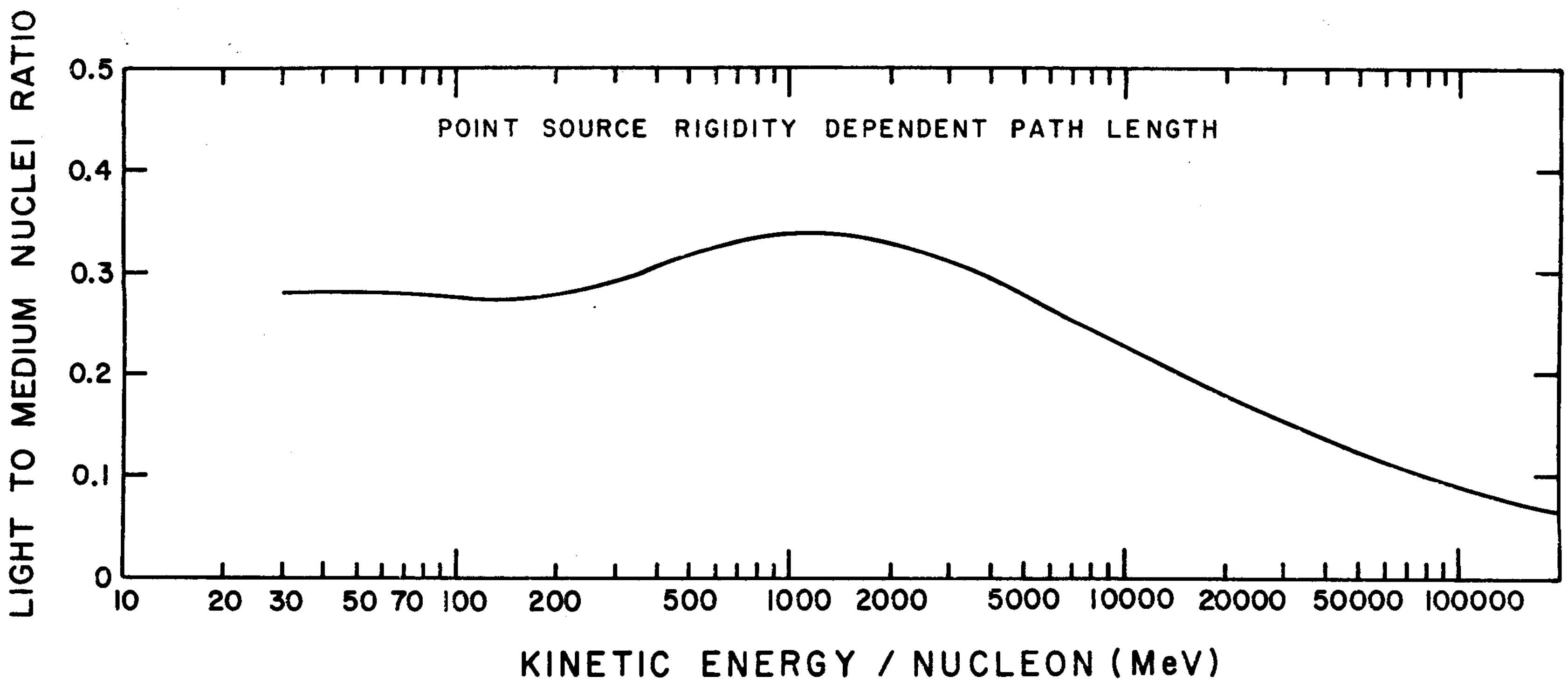
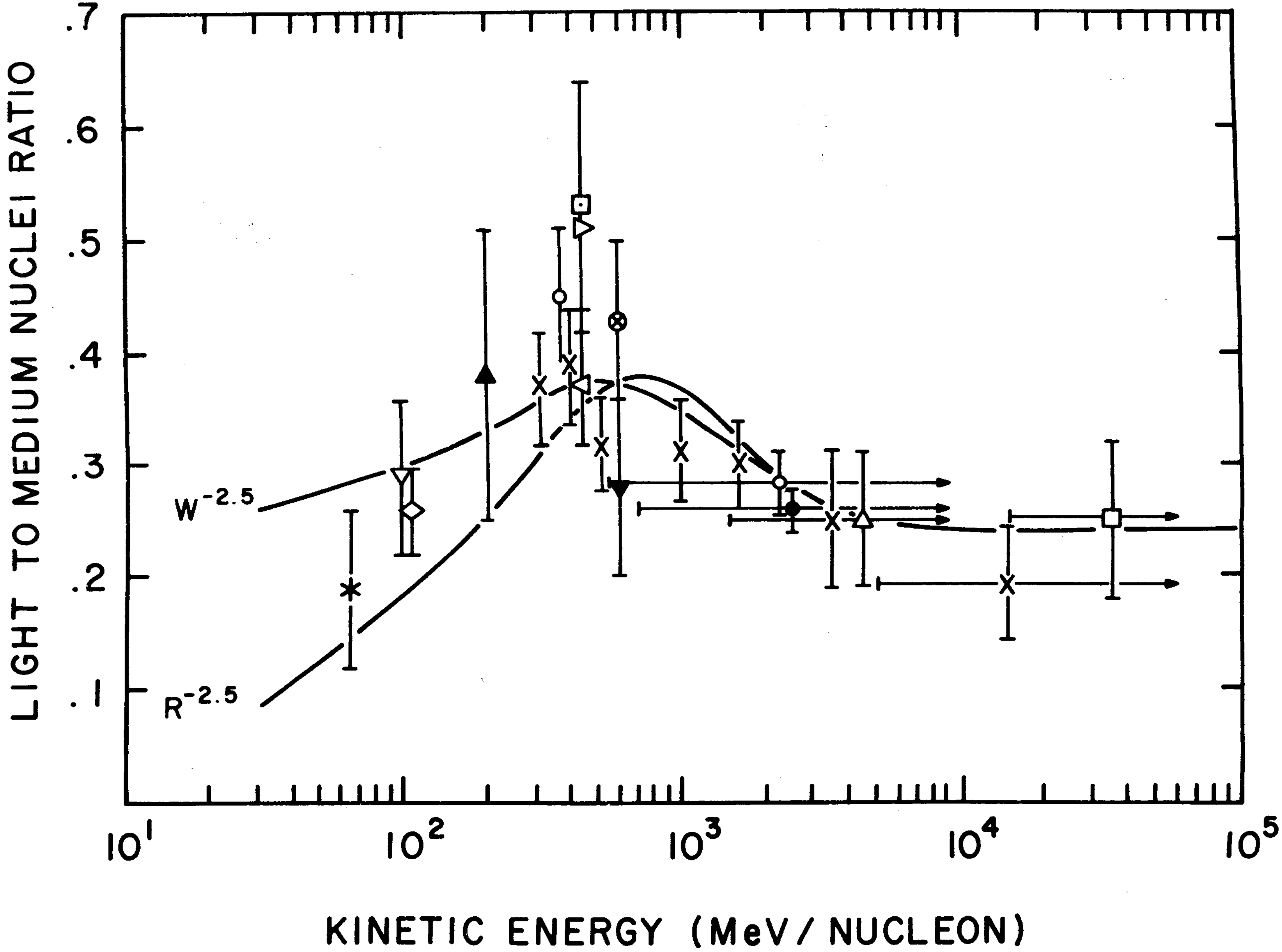
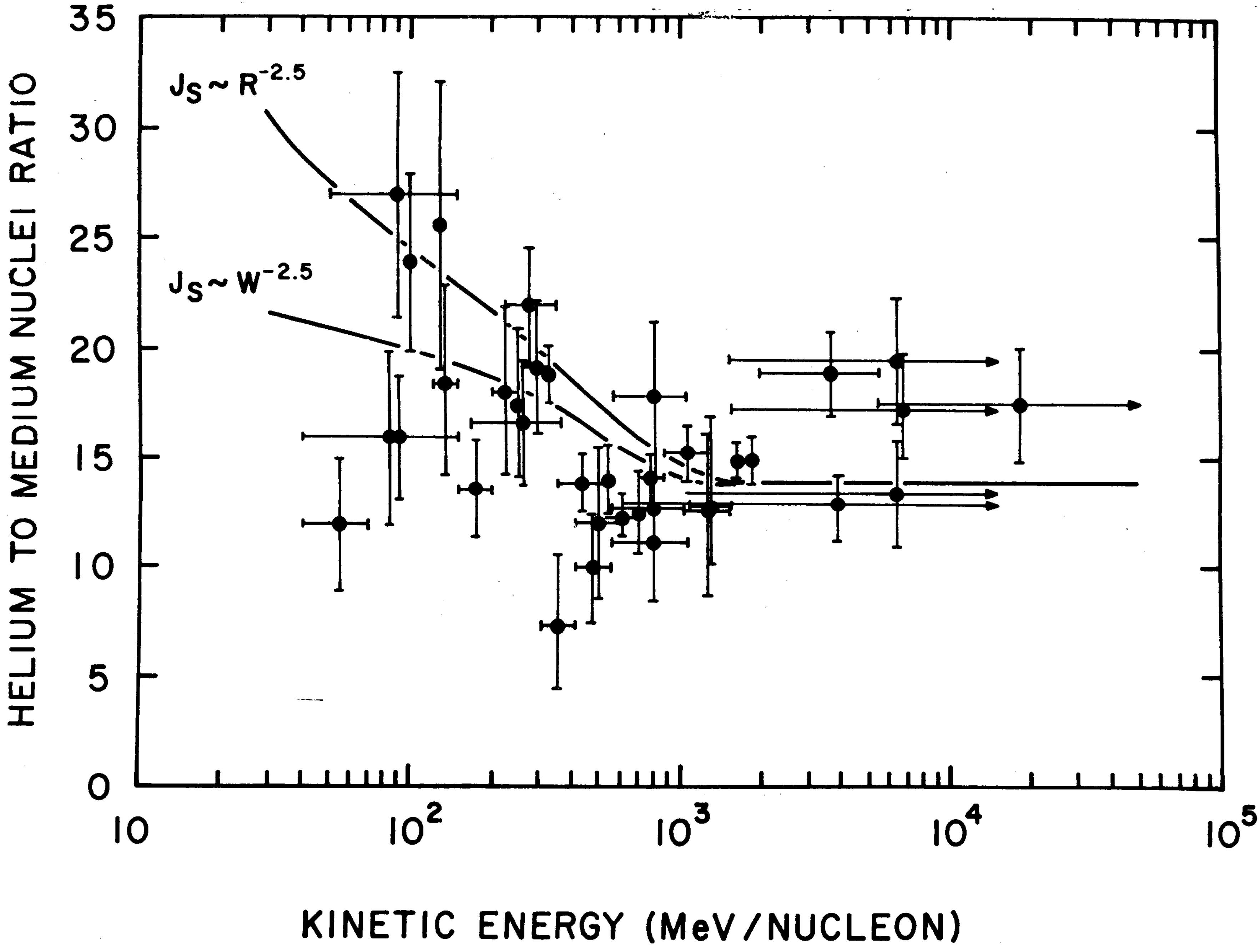

POINT SOURCE RIGIDITY DEPENDENT PATH LENGTH
LIGHT TO MEDIUM NUCLEI RATIO
0.5
0.4
0.3
0.2
0.1
0
10
20
30
50
70
100
200
500
1000
2000
5000
10000
20000
50000
100000
KINETIC ENERGY / NUCLEON (MeV)

FIGURE 11



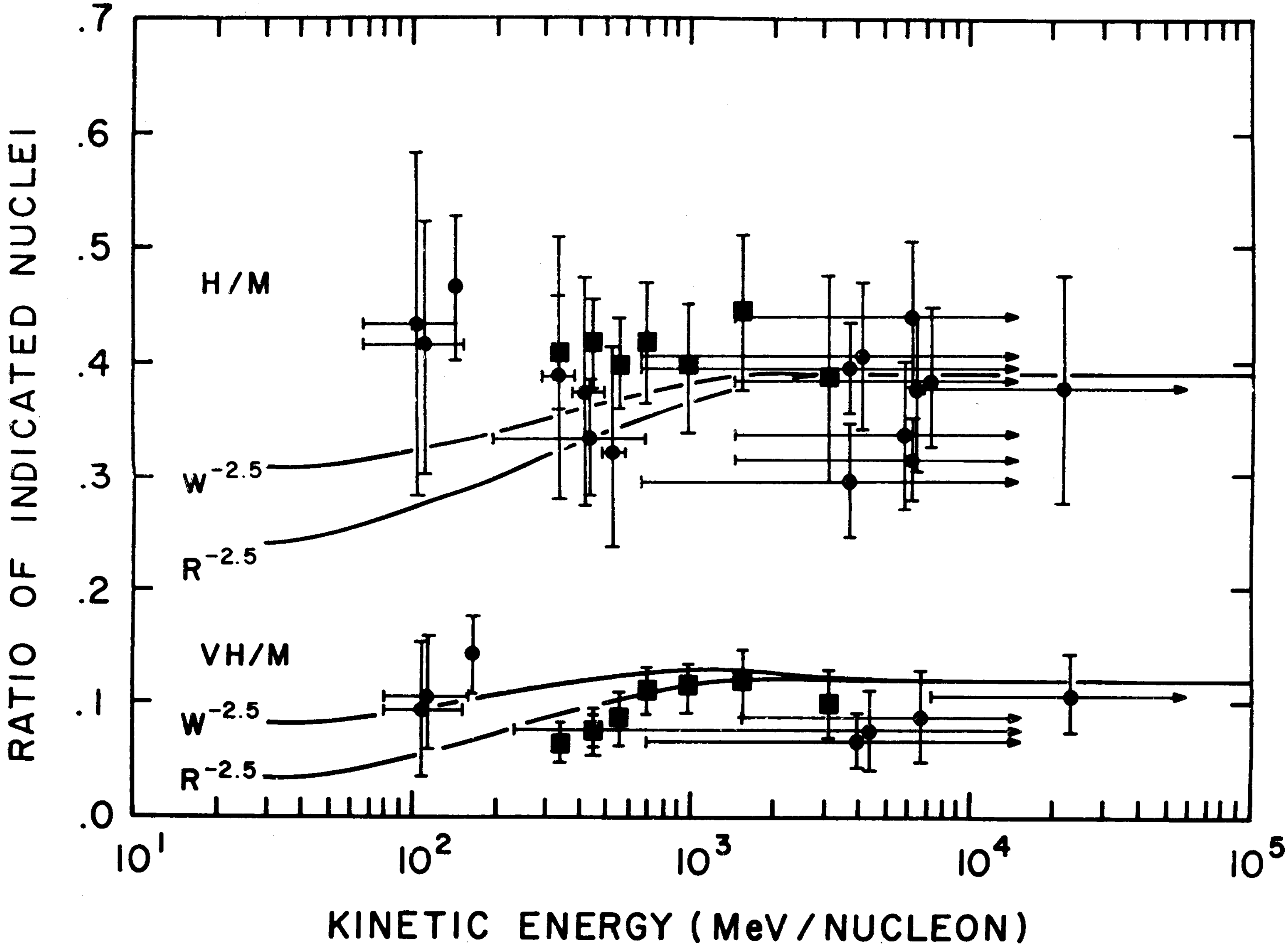
LIGHT TO MEDIUM NUCLEI RATIO
.7
.6
.5
.4
.3
.2
.1
$W^{-2.5}$
$R^{-2.5}$
10^1
10^2
10^3
10^4
10^5
KINETIC ENERGY (MeV/ NUCLEON)

FIGURE 12



HELIUM TO MEDIUM NUCLEI RATIO
35
30
25
20
15
10
5
0
$J_S \sim R^{-2.5}$
$J_S \sim W^{-2.5}$
10
10^2
10^3
10^4
10^5
KINETIC ENERGY (MeV/NUCLEON)

FIGURE 13



RATIO OF INDICATED NUCLEI
.7
.6
.5
.4
.3
.2
.1
.0
H/M
VH/M
$W^{-2.5}$
$R^{-2.5}$
$W^{-2.5}$
$R^{-2.5}$
10^1
10^2
10^3
10^4
10^5
KINETIC ENERGY (MeV/NUCLEON)

FIGURE 14